

CERN-TH.7086/93  
IP-ASTP-30-93  
hep-ph/9311327

# Subleading Corrections to Semileptonic $\Lambda_c$ Decays

**Guey-Lin Lin**

Institute of Physics, Academia Sinica, Taipei  
Taiwan 11529, Republic of China

**Thomas Mannel**

CERN – Theory Division  
CH – 1211 Geneva 23  
Switzerland

*Submitted to Phys. Lett B*

## Abstract

The semileptonic decay  $\Lambda_c \rightarrow \Lambda \ell \nu$  is considered in the framework of heavy quark effective theory beyond the leading order in the  $1/m_c$  expansion. According to our estimate the polarization variable will receive only very small corrections such that  $\alpha_{\Lambda_c} \leq -0.95$ .

# 1 Introduction

The heavy quark limit has become a standard tool for the analysis of processes involving heavy quarks [1, 2]. The additional symmetries present in the infinite mass limit yield useful constraints on the number of independent form factors parametrizing heavy hadron weak decays. Furthermore, due to the symmetries one may infer the normalization of the form factors for transitions among heavy quarks at certain kinematic points.

Corrections to the limiting behaviour may be calculated systematically in the framework of heavy quark effective field theory [2] as a power series expansion in  $\alpha_s(m_Q)$  and  $\bar{\Lambda}/m_Q$ . Here  $m_Q$  is the mass of the heavy quark and  $\bar{\Lambda}$  is a scale characterizing the light degrees of freedom.

Heavy quark symmetries yield also some predictions for the decays of heavy quarks into light ones [3]. However, in the mesonic case there is no constraint on the number of independent form factors for weak decays any more, but heavy flavor symmetry may still be used to relate semileptonic  $B$  decays to the corresponding  $D$  decays, while the heavy quark spin symmetry relates semileptonic  $b \rightarrow u$  decays to the rare  $b \rightarrow s$  decays.

From the point of view of heavy quark symmetries the simplest objects are the  $\Lambda$ -type baryons, in which the light degrees of freedom form a spin-zero state.

It has been observed that for the case of such a heavy baryon decaying into a light spin 1/2 baryon, heavy quark symmetries indeed restrict the number of independent form factors [4]. To leading order in the  $\bar{\Lambda}/m_Q$  expansion, this type of transition is described by only two form factors. In addition, these two form factors enter in such a combination that the ratio  $G_A/G_V$  turns out to be minus one. This fact has also interesting consequences for the heavy mass expansion in the decay  $\Lambda_b \rightarrow \Lambda_c \ell \nu$  [5].

This prediction, namely  $G_A/G_V = -1$ , has been confirmed by experiments. The first indirect measurement of this parameter has been performed in a nonleptonic decay, where the polarization variable  $\alpha^1$  in the nonleptonic decay  $\Lambda_c \rightarrow \Lambda \pi$  was measured. Both ARGUS [7] and CLEO [8] observe polarizations which are consistent with  $\alpha = -1$ . If factorization is assumed to relate the decay  $\Lambda_c \rightarrow \Lambda \pi$  to semileptonic  $\Lambda_c \rightarrow \Lambda$  transitions the observed value exactly corresponds to  $G_A/G_V = -1$  predicted by heavy quark

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<sup>1</sup>For the definition of the polarization variables in baryon nonleptonic decays see [6].

symmetry. However, this is a nonleptonic decay and factorization has been assumed ad hoc; hence this result may be as well accidental.

A more stringent test of this prediction requires the measurement of the  $\Lambda_c \rightarrow \Lambda$  semileptonic decay mode. Data on this has become available recently; both ARGUS [9] and CLEO [10] performed a measurement of the polarization in the semileptonic decay  $\Lambda_c \rightarrow \Lambda \ell \nu$  with results consistent with the heavy quark symmetry prediction. The errors on both measurements are still at a level of 30 percent and are thus quite large; however, these errors will be reduced in the near future.

As data becomes more precise, one has to include corrections to the heavy quark limit prediction using heavy quark effective field theory. The leading logarithmic QCD correction will not modify the result for  $G_A/G_V$  since both the axial and the vector current scale in the same way at scales below the heavy quark mass. Consequently, the leading corrections will be the ones of order  $\Lambda_{QCD}/m_c$  which may be parametrized in terms of new form factors. To some extent this analysis has been performed in [11]; however, not all the form factors allowed by heavy quark symmetries were considered in [11]. In the present paper we shall investigate the subleading corrections to the leading order result  $G_A/G_V = -1$  using the heavy quark effective theory. This amounts to expressing  $G_A/G_V$  in terms of all the leading and subleading form factors in the  $1/m_c$  expansion. It turns out that this expression involves too many form factors such that an estimate of the subleading effects requires additional input beyond the heavy quark effective theory. In the present paper we use the limit of a heavy  $s$  quark as a model assumption.

In the next section we shall give parametrizations for the leading and the subleading form factors for a general baryonic heavy to light transition and count the number of independent form factors. In section 3 we focus on transitions induced by the left handed current and extract  $G_A/G_V$  in terms of the leading and subleading form factors. Finally, we estimate the deviation to the lowest order relation  $G_A/G_V = -1$  arising from terms of the order  $1/m_c$ .

## 2 Form Factors for Baryonic Heavy to Light Transitions

It has been pointed out in [12, 4] that to leading order in the  $1/m_c$  expansion only two independent form factors  $\Phi_1$  and  $\Phi_2$  are needed to parametrize the decay of a  $\Lambda_c$  into a spin 1/2 light baryon denoted here generically as  $\Lambda$

$$\langle \Lambda(p) | \bar{q} \Gamma c_v | \Lambda_c(v) \rangle = \bar{u}(p) [\Phi_1 + \Phi_2 \not{q}] \Gamma u(v) \quad (1)$$

Here  $q$  is some light quark,  $c_v$  is the static heavy  $c$  quark and  $\Gamma$  is an arbitrary collection of Dirac matrices.

For the case of a left handed current  $\Gamma = \gamma_\mu(1 - \gamma_5)$ , the above transition can be parametrized as follows:

$$\begin{aligned} \langle \Lambda(p) | \bar{q} \gamma_\mu (1 - \gamma_5) c | \Lambda_c(v) \rangle &= \bar{u}(p) [f_1 \gamma_\mu + i f_2 \sigma_{\mu\nu} q^\nu + f_3 q^\mu] u(p') \\ &+ \bar{u}(p) [g_1 \gamma_\mu + i g_2 \sigma_{\mu\nu} q^\nu + g_3 q^\mu] \gamma_5 u(p') \end{aligned} \quad (2)$$

where  $p' = m_{\Lambda_c} v$  is the momentum of the  $\Lambda_c$  whereas  $q = m_{\Lambda_c} v - p$  is the momentum transfer. From this one defines the ratio  $G_A/G_V$  by

$$\frac{G_A}{G_V} = \frac{g_1(q^2 = 0)}{f_1(q^2 = 0)} \quad (3)$$

In the heavy  $c$  quark limit one may relate the six form factors  $f_i$  and  $g_i$  ( $i = 1, 2, 3$ ) to the two form factors  $\Phi_j$  ( $j = 1, 2$ )

$$f_1 = -g_1 = \Phi_1 + \frac{m_\Lambda}{m_{\Lambda_c}} \Phi_2 \quad (4)$$

$$f_2 = f_3 = -g_2 = -g_3 = \frac{1}{m_{\Lambda_c}} \Phi_2 \quad (5)$$

from which one reads off  $G_A/G_V = -1$ .

The  $O(1/m_c)$ -contributions to  $G_A/G_V$  originate from two sources. From the matching of the field operator of the  $c$  quark one has, to the order  $1/m_c$ , the replacement

$$c \longrightarrow c_v + \frac{1}{2m_c} i \not{D} c_v \quad (6)$$

where  $D$  is the covariant derivative of QCD acting on the  $c$  quark. This replacement will introduce local correction terms of the form

$$\Delta_l = \frac{1}{2m_c} < \Lambda(p) | \bar{q} \Gamma i \not{D} c_v | \Lambda_c(v) > \quad (7)$$

In addition to that, one also obtains contributions from the correction of the Lagrangian to order  $1/m_c$  which are given in terms of three operators

$$\begin{aligned} \mathcal{L}_1 &= \bar{c}_v \frac{(iv \cdot D)^2}{2m_c} h_c & \mathcal{L}_2 &= \bar{c}_v \frac{(iD)^2}{2m_c} c_v \\ \mathcal{L}_3 &= -\frac{g}{4m_c} \bar{c}_v \sigma_{\alpha\beta} G^{\alpha\beta} c_v \end{aligned} \quad (8)$$

These operators yield nonlocal terms of the form

$$\Delta_{nl} = i \int d^4x \sum_{j=1}^3 < \Lambda(p) | T [\mathcal{L}_j(x) \bar{q}(0) \Gamma c_v(0)] | \Lambda_c(v) > \quad (9)$$

These relations are obtained by matching at the scale of the  $c$  quark; the renormalization group flow from scaling down to smaller scales will introduce mixing of these operators with other operators of the same dimension [13, 14].

We start the discussion with the local terms and consider the matrix element

$$R_\mu = < \Lambda(p) | \bar{q} \Gamma i D_\mu c_v | \Lambda_c(v) > \quad (10)$$

which may be rewritten due to spin symmetry as

$$R_\mu = \bar{\Lambda} \bar{u}(p) \mathcal{M}_\mu \Gamma u(v) \quad (11)$$

The scale of  $R$  is set by the parameter  $\bar{\Lambda}$  defined by

$$\bar{\Lambda} = m_{\Lambda_c} - m_c = \mathcal{O}(\Lambda_{QCD}); \quad (12)$$

we have made the dependence on  $\bar{\Lambda}$  explicit in order to obtain dimensionless quantities of order one in what follows. The Dirac matrix  $\mathcal{M}_\mu$  describes the light degrees of freedom and may be expanded in terms of the sixteen independent Dirac matrices. It is constrained by the equation of motion of the heavy quark

$$v \cdot R = < \Lambda(p) | \bar{q} \Gamma (iv \cdot D) c_v | \Lambda_c(v) > = 0 \quad (13)$$

which yields  $v \cdot \mathcal{M} = 0$ . To this end,  $\mathcal{M}_\mu$  may thus be expressed in terms of four scalar functions  $A, B, C$  and  $D$

$$\mathcal{M}_\mu = (A + \not{v}B) \left( \frac{p_\mu}{v \cdot p} - v_\mu \right) + iC\sigma_{\mu\nu}v^\nu + iD\varepsilon_{\mu\alpha\beta\rho}v^\alpha \frac{p^\beta}{v \cdot p} \gamma^\rho \gamma_5 \quad (14)$$

where  $A, B, C$  and  $D$  depend on the variable  $v \cdot p$  and do not scale with the heavy mass for a fixed  $v \cdot p$ .

In fact these four form factors are not entirely independent from the leading ones. First of all, momentum conservation implies

$$\begin{aligned} i\partial_\mu < \Lambda(p) | \bar{q} \Gamma c_v | \Lambda_c(v) > &= -(p_\mu - \bar{\Lambda} v_\mu) < \Lambda(p) | \bar{q} \Gamma c_v | \Lambda_c(v) > \\ = < \Lambda(p) | -i \overline{D_\mu q} \Gamma c_v | \Lambda_c(v) > &+ < \Lambda(p) | \bar{q} \Gamma i D_\mu c_v | \Lambda_c(v) > \end{aligned} \quad (15)$$

This relation may be exploited by inserting  $\Gamma \rightarrow \gamma^\mu \Gamma$  and by contracting the index  $\mu$ . Using the equation of motion for the light quark  $i \not{D} q = m_q q$  one obtains

$$\bar{u}(p) [\Phi_1 + \not{v}\Phi_2] (\bar{\Lambda} \not{v} - \not{p} + m_q) \Gamma u(v) = \bar{\Lambda} \bar{u}(p) \mathcal{M}_\mu \gamma^\mu \Gamma u(v) \quad (16)$$

This equation leads to two relations between the leading and the subleading form factors

$$\begin{aligned} (\bar{\Lambda} - 2v \cdot p) \Phi_2 - (m_\Lambda - m_q) \Phi_1 &= \bar{\Lambda} \left( \frac{m_\Lambda}{v \cdot p} A + B - 2D \right) \\ \bar{\Lambda} \Phi_1 + (m_\Lambda + m_q) \Phi_2 &= \bar{\Lambda} \left( -A - \frac{m_\Lambda}{v \cdot p} (B - 2D) + 3C \right) \end{aligned} \quad (17)$$

The second equation allows to express  $C$  in terms of  $A$  alone, while the first equation may be used to express  $D$  in terms of  $A$  and  $B$ . We shall treat the four form factors  $\Phi_1, \Phi_2, A$  and  $B$  as independent quantities.

The nonlocal contributions will introduce more independent form factors. Due to the equation of motion of the heavy quark, the first term  $\mathcal{L}_1$  will not contribute. The second term  $\mathcal{L}_2$  is a singlet under spin symmetry and hence will yield a renormalization of the leading order form factors

$$\begin{aligned} i \int d^4x < \Lambda(p) | T [(\mathcal{L}_1(x) + \mathcal{L}_2(x)) \bar{q}(0) \Gamma c_v(0)] | \Lambda_c(v) > \\ = \frac{\bar{\Lambda}}{2m_c} \bar{u}(p) [\delta_1 + \delta_2 \not{v}] \Gamma u(v). \end{aligned} \quad (18)$$

Finally, the magnetic moment operator  $\mathcal{L}_3$  may be written as

$$i \int d^4x \langle \Lambda(p) | T [\mathcal{L}_3(x) \bar{q}(0) \Gamma c_v(0)] | \Lambda_c(v) \rangle = i \frac{\bar{\Lambda}}{2m_c} \bar{u}(p) T^{\alpha\beta} \Gamma \frac{\not{v} + 1}{2} \sigma_{\alpha\beta} u(v) \quad (19)$$

where the Dirac matrix  $T_{\mu\nu}$  introduces another five independent form factors

$$T_{\mu\nu} = -i\rho_1\sigma_{\mu\nu} + \rho_2(p_\mu\gamma_\nu - p_\nu\gamma_\mu) + i\varepsilon_{\mu\nu\alpha\beta}v^\alpha p^\beta(\rho_3 + \not{v}\rho_4)\gamma_5 + i\varepsilon_{\mu\nu\alpha\beta}v^\alpha\gamma^\beta\gamma_5\rho_5 \quad (20)$$

In the following we shall concentrate on the discussion of the left handed current and calculate the form factors  $f_i$  and  $g_i$  in terms the ones parametrizing the leading and subleading contributions.

### 3 Subleading Terms of the Left Handed Current

For the left handed current one simply takes  $\Gamma = \gamma_\mu(1 - \gamma_5)$  and re-arrange the subleading contributions according to the parametrization shown in Eq. (2).

The nonlocal contribution involving  $\mathcal{L}_2$  is a singlet under spin symmetry; hence the two form factors  $\delta_1$  and  $\delta_2$  only renormalize the leading order form factors  $\Phi_1$  and  $\Phi_2$ .

$$\tilde{\Phi}_1 = \Phi_1 + \delta_1 \frac{\bar{\Lambda}}{2m_c}, \quad \tilde{\Phi}_2 = \Phi_2 + \delta_2 \frac{\bar{\Lambda}}{2m_c}. \quad (21)$$

This result is in fact independent of  $\Gamma$ .

Furthermore, the nonlocal contribution coming from the chromomagnetic moment operator  $\mathcal{L}_2$  has in total five form factors. However, two of them,  $\rho_3$  and  $\rho_4$  only renormalize two of the form factors parametrizing the local  $1/m_c$  contributions

$$\tilde{A} = A + 2v \cdot p \rho_3, \quad \tilde{B} = B + 2v \cdot p \rho_4. \quad (22)$$

In contrast to the previous case, this relation holds only in the case of a left-handed current.

In terms of these redefined form factors one obtains for the form factors  $f_i$  and  $g_i$  the following result

$$\begin{aligned}
f_1 = & \tilde{\Phi}_1 + \frac{m_\Lambda}{m_{\Lambda_c}} \tilde{\Phi}_2 \\
& + \frac{\bar{\Lambda}}{2m_c} \left[ -\tilde{A} \left( 1 - \frac{m_{\Lambda_c}}{v \cdot p} \right) + \tilde{B} \left( \frac{m_\Lambda}{m_{\Lambda_c}} \left( \frac{m_\Lambda}{v \cdot p} - 1 \right) - 2 + \frac{m_\Lambda + m_{\Lambda_c}}{v \cdot p} \right) \right. \\
& - C \left( 1 + 2 \frac{m_\Lambda}{m_{\Lambda_c}} \right) - \frac{D}{v \cdot p} \left( m_{\Lambda_c} - \frac{m_\Lambda^2}{m_{\Lambda_c}} \right) + \rho_1 \left( 2 + 4 \frac{m_\Lambda}{m_{\Lambda_c}} \right) \\
& \left. - 2\rho_2 \left( m_{\Lambda_c} - \frac{m_\Lambda^2}{m_{\Lambda_c}} \right) - 2\rho_5 \left( 2 + \frac{m_\Lambda}{m_{\Lambda_c}} \right) \right]
\end{aligned} \tag{23}$$

$$\begin{aligned}
f_2 = & \frac{1}{m_{\Lambda_c}} \tilde{\Phi}_2 \\
& + \frac{\bar{\Lambda}}{2m_c} \left[ \tilde{A} \frac{1}{v \cdot p} + \tilde{B} \left( \frac{1}{v \cdot p} \left( \frac{m_\Lambda}{m_{\Lambda_c}} + 1 \right) - \frac{1}{m_{\Lambda_c}} \right) \right. \\
& - 2C \frac{1}{m_{\Lambda_c}} + \frac{D}{v \cdot p} \left( \frac{m_\Lambda}{m_{\Lambda_c}} - 1 \right) + 4\rho_1 \frac{1}{m_{\Lambda_c}} \\
& \left. - 2\rho_2 \left( 1 - \frac{m_\Lambda}{m_{\Lambda_c}} \right) - 2\rho_5 \frac{1}{m_{\Lambda_c}} \right]
\end{aligned} \tag{24}$$

$$\begin{aligned}
f_3 = & \frac{1}{m_{\Lambda_c}} \tilde{\Phi}_2 \\
& + \frac{\bar{\Lambda}}{2m_c} \left[ -\tilde{A} \frac{1}{v \cdot p} + \tilde{B} \left( \frac{1}{v \cdot p} \left( \frac{m_\Lambda}{m_{\Lambda_c}} - 1 \right) - \frac{1}{m_{\Lambda_c}} \right) \right. \\
& - 2C \frac{1}{m_{\Lambda_c}} + \frac{D}{v \cdot p} \left( \frac{m_\Lambda}{m_{\Lambda_c}} + 1 \right) + 4\rho_1 \frac{1}{m_{\Lambda_c}} \\
& \left. + 2\rho_2 \left( 1 + \frac{m_\Lambda}{m_{\Lambda_c}} \right) - 2\rho_5 \frac{1}{m_{\Lambda_c}} \right]
\end{aligned} \tag{25}$$

$$\begin{aligned}
g_1 = & -\tilde{\Phi}_1 - \frac{m_\Lambda}{m_{\Lambda_c}} \tilde{\Phi}_2 \\
& + \frac{\bar{\Lambda}}{2m_c} \left[ \tilde{A} \left( 1 - \frac{m_{\Lambda_c}}{v \cdot p} \right) + \tilde{B} \left( \frac{m_\Lambda}{m_{\Lambda_c}} \left( \frac{m_\Lambda}{v \cdot p} + 1 \right) - 2 - \frac{m_\Lambda - m_{\Lambda_c}}{v \cdot p} \right) \right. \\
& \left. + C \left( 1 - 2 \frac{m_\Lambda}{m_{\Lambda_c}} \right) - \frac{D}{v \cdot p} \left( m_{\Lambda_c} - \frac{m_\Lambda^2}{m_{\Lambda_c}} \right) - \rho_1 \left( 2 - 4 \frac{m_\Lambda}{m_{\Lambda_c}} \right) \right]
\end{aligned} \tag{26}$$



$$\left. -2\rho_2(m_{\Lambda_c} - \frac{m_\Lambda^2}{m_{\Lambda_c}}) - 2\rho_5(2 - \frac{m_\Lambda}{m_{\Lambda_c}}) \right]$$

$$\begin{aligned} g_2 = & -\frac{1}{m_{\Lambda_c}}\tilde{\Phi}_2 \\ & +\frac{\bar{\Lambda}}{2m_c}\left[\tilde{A}\frac{1}{v\cdot p} + \tilde{B}\left(\frac{1}{v\cdot p}\left(\frac{m_\Lambda}{m_{\Lambda_c}} - 1\right) + \frac{1}{m_{\Lambda_c}}\right)\right. \\ & -2C\frac{1}{m_{\Lambda_c}} + \frac{D}{v\cdot p}\left(\frac{m_\Lambda}{m_{\Lambda_c}} + 1\right) + 4\rho_1\frac{1}{m_{\Lambda_c}} \\ & \left.+ 2\rho_2\left(1 + \frac{m_\Lambda}{m_{\Lambda_c}}\right) + 2\rho_5\frac{1}{m_{\Lambda_c}}\right] \end{aligned} \quad (27)$$

$$\begin{aligned} g_3 = & -\frac{1}{m_{\Lambda_c}}\tilde{\Phi}_2 \\ & +\frac{\bar{\Lambda}}{2m_c}\left[-\tilde{A}\frac{1}{v\cdot p} + \tilde{B}\left(\frac{1}{v\cdot p}\left(\frac{m_\Lambda}{m_{\Lambda_c}} + 1\right) + \frac{1}{m_{\Lambda_c}}\right)\right. \\ & -2C\frac{1}{m_{\Lambda_c}} + \frac{D}{v\cdot p}\left(\frac{m_\Lambda}{m_{\Lambda_c}} - 1\right) + 4\rho_1\frac{1}{m_{\Lambda_c}} \\ & \left.- 2\rho_2\left(1 - \frac{m_\Lambda}{m_{\Lambda_c}}\right) + 2\rho_5\frac{1}{m_{\Lambda_c}}\right] \end{aligned} \quad (28)$$

These expressions are quite lengthy and one has to point out that after redefinitions of  $\Phi_1$ ,  $\Phi_2$ ,  $A$  and  $B$  there are in total nine independent form factors parametrizing the leading and subleading contributions. Note that one can no longer apply (17) to eliminate  $C$  and  $D$  since one has already redefined  $\Phi_1$ ,  $\Phi_2$ , etc. It is well known that the most general parametrization of the left handed current needs only six form factors; hence the heavy quark limit alone will not be useful once the subleading corrections are taken into account. In order to proceed further one needs additional input beyond heavy quark symmetry which will be discussed in the next section.

## 4 Discussion of the Result

Some insight into the anatomy of the subleading corrections may be obtained by considering formally the infinite mass limit also for the quark  $q$ . In that case all the form factors which violate the spin symmetry of the quark  $q$

vanish in the leading order. This implies that the form factor  $\Phi_2$  should be excluded from the leading-order structure. For local  $1/m_c$ -corrections, only the form factor  $A$  is allowed by spin symmetry; all other form factors are suppressed by  $1/m_q$ . In addition, it has been shown in [15] that the subleading form factor  $A$  may be related to the leading one  $\Phi_1$ . In the present formalism this result may be obtained by observing that in the heavy mass limit for the quark  $q$ , one has, to the leading order in the  $1/m_q$  expansion,  $m_{\Lambda_c} - m_Q = m_\Lambda - m_q = \bar{\Lambda}$ . Furthermore, the spin symmetry of  $q$ , (15) implies the substitution  $\Gamma \rightarrow \frac{1+\not{v}}{2}\Gamma$ , where  $\not{v}$  is the four velocity of  $q$ . Due to the projection  $\frac{1+\not{v}}{2}$   $\not{v}$  in (16) becomes replaced by  $v \cdot v'$  and the two relations of (17) collapse into a single one which is

$$\Phi_1(v \cdot v' - 1) = \frac{A}{v \cdot v'}(1 - (v \cdot v')^2). \quad (29)$$

This exactly reproduces the result obtained in [15].

However, as it was shown in [5], the linear terms in both  $1/m_c$  as well as  $1/m_q$  do not change the relation  $f_1 = -g_1$  and thus one still has  $G_A/G_V = -1$  to this order. Terms, which change this relation, will thus be of the order of  $\bar{\Lambda}^2/(m_c m_q)$  in a framework, where an expansion in inverse powers of the mass  $m_q$  makes sense. Terms of this kind have been studied in [16]; however, for a quantitative estimate of these terms, an input beyond the heavy quark effective theory is necessary.

In [16] the ratio  $G_A/G_V$  is studied for the case of  $\Lambda_b \rightarrow \Lambda_c e \nu$ . In this case the ratio  $G_A/G_V$  depends on three unknown parameters which are ratios of form factors, taken at the point  $q^2 = 0$ . The corrections in this case have been estimated to be of the order of a few percent, thus being of the typical order of magnitude expected from the quantity  $\bar{\Lambda}^2/(m_c m_b) \sim 1\%$ . It has been pointed out [16] that the quantity which is accessible experimentally is the polarization variable  $\alpha_{\Lambda_c}$ . The relation of this variable to the ratio of form factors is given by

$$\alpha_{\Lambda_c} = \frac{2x}{1+x^2} \text{ where } x = G_A/G_V \quad (30)$$

This variable is quite insensitive to corrections to  $G_A/G_V$  if  $G_A/G_V$  is close to  $-1$ : If  $x = -1 + \epsilon$ , then  $\alpha_{\Lambda_c}$  receives only corrections of the order  $\epsilon^2$

$$\alpha_{\Lambda_c} = -1 + \frac{1}{2}\epsilon^2 \quad (31)$$

This means that the polarization variable only receives corrections of the order  $1/m_c^2$ , although the form factors and their ratios will have corrections linear in  $1/m_c$ .

Another very rough estimate on the corrections to the polarization variable may be obtained by using the information of the exclusive semileptonic  $\Lambda$  decays. From the heavy quark arguments displayed above one would expect that  $\alpha_\Lambda + 1$  scales like  $\bar{\Lambda}^2/m_s^2$  in a limit, where the  $s$  quark is heavy. In order to obtain an estimate for  $\alpha_{\Lambda_c}$  we shall take this limit as a model assumption; we may then use the measured value of  $\alpha_\Lambda$  in the decay  $\Lambda \rightarrow p e \nu$  to scale up to the value of  $\alpha_{\Lambda_c}$

$$0.282 \pm 0.015 = (\alpha_\Lambda + 1) = \frac{m_c^2}{m_s^2}(\alpha_{\Lambda_c} + 1) \quad (32)$$

where the measured value has been taken from [6]. Taking a value of 1.8 GeV for the  $c$  quark mass and varying the mass of the  $s$  quark in the range between 400 and 600 MeV one obtains for the corrections to the polarization variable

$$0.01 \leq (\alpha_{\Lambda_c} + 1) \leq 0.04 \quad (33)$$

and hence one expects a fairly small deviation from unity. However, the smallness of the correction is mainly due to the fact that the polarization variable  $\alpha$  is insensitive to corrections to  $G_A/G_V = -1$ . If one translates this estimate for  $\alpha_{\Lambda_c}$  into a result for  $G_A/G_V$  one has

$$0.14 \leq |G_A/G_V + 1| \leq 0.25 \quad (34)$$

This result is consistent with the estimate given for  $\Lambda_b \rightarrow \Lambda_c e \nu$  in [16] if one scales up the result there by a factor of  $m_b/m_s \sim 10$ . We have to stress again that this is not meant to be a detailed quantitative analysis, however, the qualitative agreement with the scaled up result of [16] and the result obtained from the semileptonic  $\Lambda$  decays give a consistent picture of the corrections to be expected.

Finally we want to comment on perturbative QCD corrections. They have been calculated and may be found in the literature [13, 14]. The leading order result at the level of the leading logarithmic approximation (LLA) is independent of the Dirac structure of the current as a consequence of spin symmetry [2]; hence  $G_A/G_V$  is not changed in LLA.

To order  $1/m_c$ , perturbative QCD introduces additional operators; the matrix elements of these may be related to the form factors defined above and hence these corrections will not introduce additional unknown functions. However, we have not given these QCD corrections here, since they will be small, of the order  $\alpha_s(m_c)$  and  $(m_s/m_c)\alpha_s(m_c)\ln(m_c^2/m_s^2)$ , and a quantitative estimate of the effects is difficult anyway.

## 5 Conclusion

In the present paper we have given a detailed discussion on the leading recoil corrections for the case of a heavy  $\Lambda_c$  decaying into a light baryon. The prototype of such a transition is the decay  $\Lambda_c \rightarrow \Lambda e \nu$  which has recently been measured by both ARGUS and CLEO.

The main prediction of heavy quark effective theory for this class of decays is  $G_A/G_V = -1$  which has been checked in the recent experiments. It turns out that the prediction indeed holds, although the errors are still quite large.

Subleading corrections of the order of  $\bar{\Lambda}/m_c$  will change this result; however, it turns out to be difficult to estimate this change quantitatively, since in total nine unknown form factors enter the game at the order  $\bar{\Lambda}/m_c$  and thus not much can be said from the heavy quark effective theory alone.

A rough estimate may be obtained from the assumption that the  $s$  quark is heavy; however, we want to stress that this has the character of a model assumption. We expect this to at least yield the correct magnitude of the subleading terms, since the assumption of a heavy  $s$  quark has in fact yield reasonable results, once the leading recoil corrections had been taken into account [17].

From these estimates we obtain a correction between 20% and 30% for the ratio  $G_A/G_V$  which is of the expected size. However, the experimentally accessible quantity is the polarization variable  $\alpha_{\Lambda_c}$  which depends only weakly on the corrections to  $G_A/G_V$ . For this variable it is very likely that its value is smaller than  $-0.95$ . Since  $\alpha_{\Lambda_c}$  cannot exceed  $-1$ , a check of this requires a measurement of  $\alpha_{\Lambda_c}$  to the precision of a few per cent. However, this will require a reduction of the error of the present experimental data by a factor of ten, which will be difficult to achieve.

## Acknowledgements

T.M. acknowledges clarifying discussions with M. Neubert and D. Cassel. G.L.L. appreciate the hospitality of the Institut für Kernphysik, Darmstadt (Germany) where a part of this work was done. G.L.L. is supported in part by the National Science Council of the Republic of China, under the contract number NSC83-0208-M-001-014.

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